nag_arccosh (s11acc)

1. Purpose

nag_arccosh (s11acc) returns the value of the inverse hyperbolic cosine, $\operatorname{arccosh} x$. The result is in the principal positive branch.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_arccosh(double x, NagError *fail)
```

3. Description

The function calculates an approximate value for the inverse hyperbolic cosine, $\operatorname{arccosh} x$. It is based on the relation

$$\operatorname{arccosh} x = \ln(x + \sqrt{x^2 - 1}).$$

This form is used directly for $1 < x < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $x \geq 10^k$, $\sqrt{x^2 - 1}$ is equal to $\sqrt{x}$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arccosh} x = \ln 2 + \ln x.$$

4. Parameters

x

Input: the argument x of the function.
Constraint: $\mathbf{x} \geq 1.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LT

On entry, **x** must not be less than 1.0: $\mathbf{x} = \langle value \rangle$.
$\operatorname{arccosh} x$ is not defined and the result returned is zero.

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{x^2 - 1} \operatorname{arccosh} x} \delta \right|.$$

That is the relative error in the argument is amplified by a factor at least

$$\frac{x}{\sqrt{x^2 - 1} \operatorname{arccosh} x}$$

in the result. The equality should apply if δ is greater than the ***machine precision*** (δ due to data error etc.), but if δ is simply a result of round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation and round-off.

It should be noted that for $x > 2$ the factor is always less than 1.0. For large x we have the absolute error E in the result, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***. More significantly for x close to 1, $x - 1 \sim \delta$, the above analysis becomes inapplicable due to the fact that both function and argument are bounded, $x \geq 1$, $\operatorname{arccosh} x \geq 0$. In this region we have

$$E \sim \sqrt{\delta}.$$

That is, there will be approximately half as many decimal places correct in the result as there were correct figures in the argument.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.6 p 86.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_arccosh(s11acc) Example Program
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double  x, y;

  Vprintf("s11acc Example Program Results\n");
  Vscanf("%*[^\n]");       /* skip the first input line */
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s11acc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s11acc Example Program Data
    1.00
    2.0
    5.0
   10.0
```

8.3. Program Results

```
s11acc Example Program Results
     x            y
   1.000e+00   0.000e+00
   2.000e+00   1.317e+00
   5.000e+00   2.292e+00
   1.000e+01   2.993e+00
```